Towards Robust and Efficient Routing in Multi-Radio, Multi-Channel Wireless Mesh Networks: Technical Report

Jonathan Wellons, Yuan Xue
Department of Electrical Engineering and Computer Science, Vanderbilt University
Email: {jonathan.wellons, yuan.xue}@vanderbilt.edu

APPENDIX: PROOF OF THEOREM 1.

Theorem 1: Let $\theta(\Phi, D) = \max_{\boldsymbol{d}^k \in D} \theta(\Phi, \boldsymbol{d}^k)$. Further let Φ_D^{opt} be the optimal solution to $\min_\Phi \theta(\Phi, D)$ and $\theta^{opt}(D)$ be the optimal value. Then for $\forall \boldsymbol{d}' \in \mathcal{D}$, $\theta(\Phi_D^{opt}, \boldsymbol{d}') \leq \theta^{opt}(D)$.

Proof: Without loss of generality, assume $\zeta = 1$. When the routing Φ_D is applied to a network under demand d, say the congestion is $\theta_D(d)$. We have

$$\theta_D(d) = \max_{\boldsymbol{c},\boldsymbol{e}} \sum_{a \in I(e)} \sum_{f \in F} d_f \phi_f^c(a)$$

We will prove **Theorem I** by contradiction. Assume there exists a point d^m in the interior of D which has congestion under Φ_D greater than any of the vertices of D.

First, because d_m is interior to D, we can write

$$d^m = \sum_i t_j d^i \text{ for some } t_i \text{ with } \sum_i t_i = 1$$

By the assumption

$$\forall i \theta_D(d^m) > \theta_D(d^i)$$

for some c and e, we have

$$\theta_D(d^m) = \sum_{a \in I(e)} \sum_{f \in F} d_f^m \phi_f^c(a)$$

and for this c and e

$$\theta_D(d^i) \geq \sum_{a \in I(e)} \sum_{f \in F} d_f^i \phi_f^c(a)$$

Thus, we can write,

$$\sum_{a \in I(e)} \sum_{f \in F} d_f^m \phi_f^c(a) > \sum_{a \in I(e)} \sum_{f \in F} d_f^i \phi_f^c(a)$$
$$\sum_{f \in F} d_f^m \sum_{a \in I(e)} \phi_f^c(a) > \sum_{f \in F} d_f^i \sum_{a \in I(e)} \phi_f^c(a)$$

Letting $b_f = \sum_{a \in I(e)} \phi_f^c(a)$, we have

$$\sum_{f \in F} d_f^m b_f > \sum_{f \in F} d_f^i b_f$$
$$t_i \sum_{f \in F} d_f^m b_f > t_i \sum_{f \in F} d_f^i b_f$$

Now summing both sides over i, we have

$$\sum_i t_i \sum_{f \in F} d_f^m b_f > \sum_i t_i \sum_{f \in F} d_f^i b_f$$
$$\sum_i t_i \left(\sum_{f \in F} d_f^m b_f \right) > \sum_{f \in F} b_f \sum_i t_i d_f^i$$
$$\sum_i t_i \left(\sum_{f \in F} d_f^m b_f \right) > \sum_{f \in F} b_f d_f^m$$
$$\sum_i t_i > 1$$

Which is a contradiction. ∎