DECENTRALIZED CONTROL OF STRUCTURAL ACOUSTIC RADIATION

Kenneth D. Frampton, PhD.,
Vanderbilt University
2400 Highland Avenue
Nashville, TN 37212
(615) 322-2778
(615) 343-6687 Fax
ken.frampton@vanderbilt.edu

ABSTRACT

Although the application of active control to vibrations has been investigated from many years, the extension of this technology to large-scale systems has been thwarted, in part, by an overwhelming need for computational effort, data transmission and electrical power. This need has been overwhelming in the sense that the potential applications are unable to bear the power, weight and complex communications requirement of large-scale centralized control systems. Recent developments in MEMS devices and networked embedded devices have changed the focus of such applications from centralized control architectures to decentralized ones. A decentralized control system is one that consists of many autonomous, or semi-autonomous, localized controllers called nodes, acting on a single plant, in order to achieve a global control objective. Each of these nodes has the following capabilities and assets: 1) a relatively limited computational capability including limited memory, 2) oversight of a suite of sensors and actuators and 3) a communications link (either wired or wireless) with neighboring or regional nodes. The objective of a decentralized controller is the same as for a centralized control system: to maintain some desirable global system behavior in the presences of disturbances. However, decentralized controllers do so with each node possessing only a limited amount of information on the global systems response. Exactly what information each node has access to, and how that information is used, is the topic of this investigation.

INTRODUCTION

Although the application of active control structural acoustic control has been investigated from many years, the extension of this technology to large-scale systems has been thwarted, in part, by an overwhelming need for computational effort, data transmission and electrical power. This need has been overwhelming in the sense that the potential applications are unable to bear the power, weight and complex communications requirement of large-scale centralized control systems. Recent developments in MEMS devices and networked embedded technologies have changed the focus of such applications from centralized control architectures to decentralized ones.

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The field of decentralized control has been the topic of numerous investigations for over 30 years[1]. Most of these studies have considered "weakly connected" systems or architectures wherein each node only experiences a few of the degrees of freedom of the entire system while being weakly connected to other parts of the system. Robotic swarms are a good example of weakly connected systems and have been the topic of many research projects in recent years. Decentralized control has been considered in a few vibration control projects for application in space structures[2,3,4] although no investigations have considered its application to structural acoustic control.

Hierarchical decentralized approaches have also been considered for control of buckling in beams as well[5].

The work reported in this manuscript was inspired by, and is and extension of the work in References 3,4 and 5. It builds on the work of How et al by considering more extensive hierarchies. The current work also extends the work of Hogg and Huberman by introducing hierarchies for control of continuous dynamic systems. Another unique feature of this work is that it considers the decentralized control by localized controllers all of whom experience the complete dynamics of the system to be controlled. That is to say that each node "sees", or obtains sensor information, which contains contributions from all of the global systems states (or degrees of freedom). This is in contrast to decentralized control of robotic swarms wherein each node (robot) only experiences it's own dynamics with weak coupling to its neighbors.

This work specifically addresses the decentralized control of structural acoustic radiation from a simply supported beam. The beam is equipped with numerous sensors and actuators. Decentralized compensators are designed which interact with each other by sharing sensor and actuator information as defined by hierarchical organization. The performance of these hierarchical, decentralized control approaches are evaluated by comparing their performance with that of a centralized control system employing the same sensors and actuators and expending an equal amount of control energy. The discussion begins with a general discussion of hierarchical, decentralized control. This is followed by the development of a specific example; namely a simply supported beam. This includes beam modeling, control design methodology and hierarchical organization. Finally, results are presented which demonstrate the effectiveness of various hierarchies in active structural acoustic control.

HIERARCHIES DECENTRALIZED CONTROL

The thrust of this work is to investigate the effectiveness of various hierarchies for decentralized structural acoustic control. In this context, hierarchies consist of layered abstract entities, called (borrowing terminology from the computer science realm) agents. These agents are software entities separate from the hardware entities, called nodes that host them. Each node may host more than one agent. These agents receive information from other agents and/or directly from sensors. Behaving like an independent controller, each agent processes it's inputs in a continuous manner to produce command outputs. This output is either passed on to another agent and/or directly to an actuator. The hierarchy is the organizational structure that defines how input/output information is shared among agents. An example of such a hierarchy is shown in Figure 1.

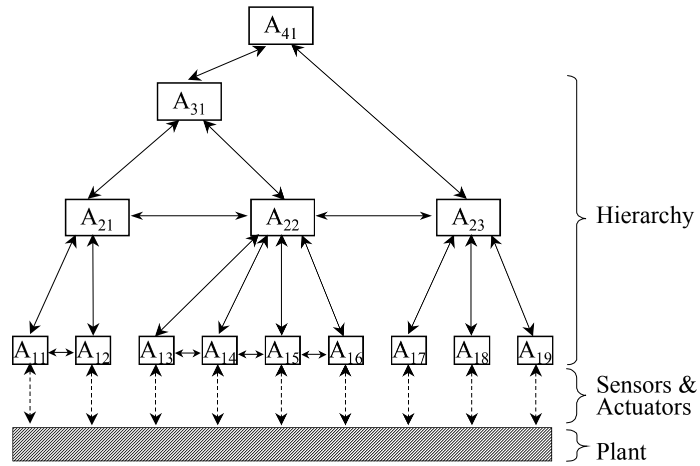


Figure 1 Schematic of decentralized, hierarchical control system.

In this hierarchy, the bottom tier of agents (i.e. A_{11}, A_{12}, …, A_{19}) are linked directly to the sensors and actuators that provide for feedback control. In general, these lowest tier agents may command a heterogeneous suite of sensors and actuators. A practical application for such a hierarchy would consist of numerous nodes distributed over the system to be controlled. Each node would host one or more agents and support the input/output needs of these agents.

In general, agents, and the hierarchies in which they reside, can be of any form. Agents may be any type of compensator such as constant gain, dynamic, adaptive or nonlinear. The hierarchies may also be of any form one can imagine. However, in order to develop agents that can be hosted by processors of limited computational ability, only simplified hierarchies and agents will be considered here. These hierarchies will be limited to regular ones; defined as hierarchies wherein each agent of a layer has similar connectivity. Furthermore, the agents will be limited to constant gain output feedback regulators.

A capability that is particularly important in active structural acoustic control is the ability of a control system to observe, and attenuate specific structural modes (i.e. those that are most efficient radiators). This is particularly challenging in a decentralized computational environment where specific nodes only have access to a limited amount of localized information. One very promising technique for constructing global modal observers in a decentralized environment is through vertical hierarchies.

A purely vertical hierarch is one that consists of numerous layers of agents that communicate only with agents above or below them in the hierarchy (no communication with agents in the same layer). An example of a vertical hierarchy is shown in Figure 2. In general, the information exchanged may be filtered sensor data, filtered control signal data, or a combination of both.

For the purposes of this study, each agent is a constant gain output feedback regulator. The design of these regulators will be discussed in the following section. Each agent then generates a single control output that is proportional to the input. This control signal is sent to all agents below it in the hierarchy (as depicted in Figure 2). Command signals received by an agent from higher-level agents are added to the command

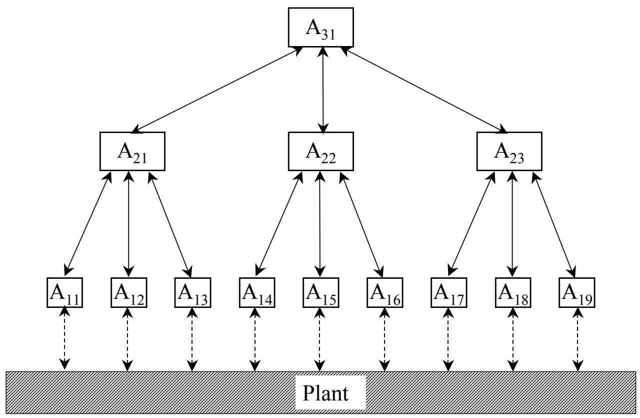


Figure 2 Schematic of a vertical hierarchy.

signal produced by that agent. This total command signal is then passed down to all lower level agents. At the lowest hierarchical level, the command signal is sent to the actuator and consists of the sum of all command signals coming from agents above the actuator in the hierarchy. The input to the lowest level agents is a single sensor signal. All upper level agents receive the average of all lower level agents inputs as their inputs. Therefore, the lone agent at the top of the hierarchy has the average of all sensor signals as inputs. So each agent has a single input that is multiplied by a single gain to produce the control output.

Two parameters of the vertical hierarchies are considered here. First is the depth of the hierarchy (i.e. the number of layers) and the second is the relative performance penalty used in each layer. When each agent is designed a performance penalty is associated with the design (see below). Thus, the design of each agent may be performed with different penalties on the system performance. The use of different performance penalties is addressed here, but is limited such that all agents on the same layer have the same penalty. Agents on different layers may have different penalties. This approach allows for some interesting properties in the control system performance as will be demonstrated.

SYSTEM MODELING AND DESIGN

The objective of this work is to investigate the effectiveness of decentralized control for active structural acoustic control. This will be demonstrated with a simply supported beam as the radiating structure. The objective of the decentralized controller is to minimize the beam response to a disturbance input by applying various hierarchical control architectures through collocated point force/point velocity feedback. The beam model and decentralized controller design are described followed by a discussion of the results obtained with various hierarchies.

Plate Dynamics

The plant under consideration is a simply supported beam subject to a point force disturbance and to point force actuation collocated with point velocity sensing. The beam is modeled using Galerkin's technique to discretize the linear equations of motion. The partial differential equation of motion is

$$0 = EI\nabla^4 w(x,t) + \rho h \frac{\partial^2 w(x,t)}{\partial t^2} + f_d(x,t) + \sum_{k=1}^{K} f_c(x,t) \quad (1)$$

where $w(x,t)$, E, I, ρ and h are the beam displacement, modulus of elasticity, density and thickness respectively. The beam is acted upon by a disturbance force, f_d, and K control forces, f_c. A separable solution is assumed using the *in vacuuo* beam eigenfunctions and generalized coordinates of the form

$$w(x,t) = \sum_{n=1}^{N} \psi_n(x) q_n(t) = \sum_{n=1}^{N} \sin\left(\tfrac{n\pi x}{L}\right) q_n(t) \quad (2)$$

where, $\Psi_n=\sin(n\pi x/L)$ are the mode shapes and $q_n(t)$ are the generalized coordinates. Substituting Eq. (2) into Eq. (1), multiplying by an arbitrary expansion function, $\Psi_m(x,y)$, and integrating over the domain yields a set of ordinary differential equations of the form:

$$0 = M_n \ddot{q}_n(t) + K_n q_n(t) + \sum_{k=1}^{K} Q_{kn}^c(t) + Q_n^d(t) \quad (3)$$

where M_n and K_n are the modal mass and stiffness and Q^c_{kn} and Q^d_n are the control generalized forces and the disturbance generalized forces. The beam model can be cast in state variable form as follows[6]:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned} \quad (4)$$

where **x** is the state vector containing the generalized coordinates, $q_n(t)$ and their derivatives, **u** is a vector of control and disturbance forces, and **y** is the beam vibrational velocity at each control point.

Agent Design: Output feedback for decentralized control

For the purpose of this investigation, hierarchies are considered which are composed of relatively simple agents. Each of these agents is an optimally designed output feedback compensators. Those agents that occupy the lowest hierarchical levels receive one or more sensor signals as inputs. Agents occupying higher hierarchical levels receive the average of all point velocity measurements coming from sensors below them in the hierarchy. Thus, all agents above the lowest level are single input compensators. The lowest level agents may have one, or multiple inputs depending on whether they are part of a horizontal hierarchy or not.

All agents are optimal, constant gain, output feedback compensators and all are designed in the same manner. Since the outputs considered hear are from point velocity sensors (or averages of point velocity measurements), the output feedback control amounts to rate feedback which has desirable stability and robustness properties[7] .

Output feedback consists of feeding back a set of measured system outputs through a constant gain compensator and back to the system as control inputs. In this case, the system output consists of point velocity sensors collocated with the point force actuators. Therefore, the control forces are based on system output such that

$$\mathbf{u}_{\mathbf{c}} = -\mathbf{K}\mathbf{y} \tag{5}$$

where $\mathbf{u}_c$ is that portion of the system input corresponding to the control forces and **K** is the feedback gain matrix and, **y** is the vector of system outputs containing point velocity measurements at each node. The feedback gain matrix can be found by minimizing the cost functional as described by Levine and Athans as[8]

$$J = \int_{t_0}^{t_\infty} \left[\mathbf{y}^{\mathbf{T}}\mathbf{Q}\mathbf{y} + \mathbf{u}_{\mathbf{c}}^{\mathbf{T}}\mathbf{R}\mathbf{u}_{\mathbf{c}} \right] dt \tag{6}$$

where **Q** is a semi-positive definite performance weighting and **R** is a positive definite control effort penalty. Details concerning the calculation of a feedback gain matrix that minimizes equation (6) can be found in Reference 8. The weighting matrix **R** was set equal to 1 in all cases (a scalar since all agents produce one output). The output weighting matrix, **Q**, was set equal to the identity matrix of appropriate dimension multiplied by a scalar whose magnitude will be discussed shortly.

$$\mathbf{Q} = \alpha \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \tag{7}$$

Each agent was designed independently based on the open loop plant and employing the method outlined previously. Once each agent was designed, all agents were appropriately connected to the open loop plant. A new system arrangement was constructed which had the closed loop system control signals as outputs and the disturbance as the only input. The H_2-norm of this system was calculated. If this norm was not equal to one, then the scalar multiple of the output weighting matrix, α, was adjusted. All agents were redesigned and the process was repeated. This iteration was continued until an acceptable accuracy was achieved.

There were two reasons for iteratively calculating the agents. The first reason was to ensure a fair comparison basis for different hierarchies. The quantity being preserved among all systems is the H_2-norm between disturbance input and control signal output. This quantity is proportional to the total energy contained in all control signals[6]. Therefore, if all hierarchies have the same H_2-norm then they will expend an equal amount of control energy. In the specific case of velocity feedback, since the loop is closed between velocity and force, this also implies that the total control power is equal.

The second reason for iteratively solving for each agent was to ensure a control system design that one could reasonably expect to achieve in an experimental setting. If the $\mathcal{H}_2$-norm between disturbance and control is set equal to one, this means that the total energy expended by the control system is equal to the disturbance energy[6]. Furthermore, this level of control effort has been found to be a reasonably achievable goal, if somewhat optimistic, for experimental implementations of control systems[9].

HIERARCHY PERFORMANCE COMPARISON

In this section, the ability of various hierarchies to target specific beam modes is investigated. Three specific vertical hierarchies are considered: 2-layer, 4-layer, and 6-layer as shown in Figure 3. In all cases, the plant to be controlled consists of a simply supported beam with 32 collocated sensors and actuators. These are equally spaced along the length of the beam. The metric used to evaluate system performance is the singular value plot of the system frequency response between the disturbance input and all sensor outputs. Furthermore, as a basis of comparison, the performance of a centralized compensator will be shown. This compensator consists of an optimally designed, constant-gain, output feedback regulator that utilizes all sensors and actuators while expending the same amount of total energy as the decentralized compensators. Two parameters of the vertical hierarchies are considered. First is the depth of the hierarchy (i.e. the number of layers) and the second is the relative performance penalty, α, used in each layer.

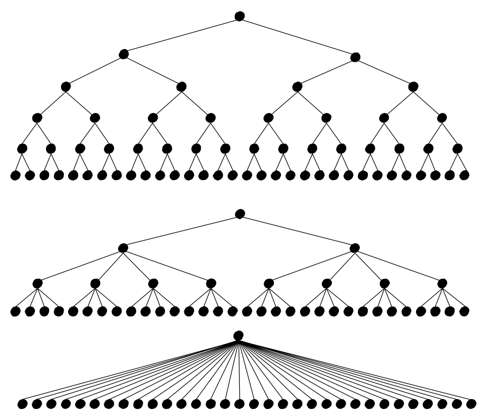


Figure 3 Diagram of the 2-layer, 4-layer and 6-layer vertical hierarchies.

A comparison of the performance of hierarchies with various numbers of layers is shown if Figure 4. In this case, the

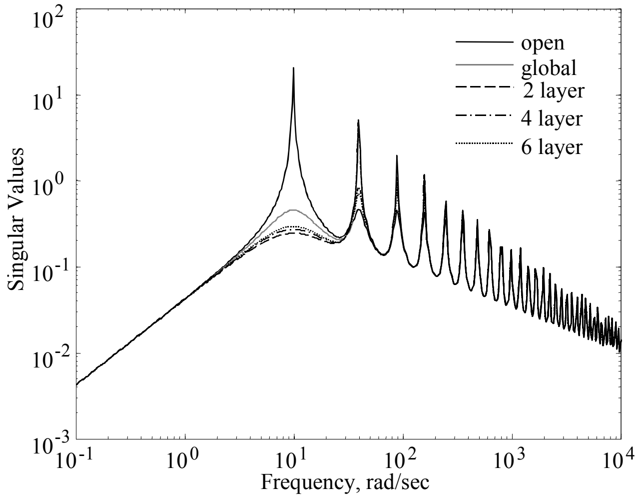


Figure 4 Performance comparison of various vertical hierarchies.

same performance penalty weight, α, was used to design all agents in the hierarchy. Note that for the lowest natural frequency, all of the hierarchies were able to outperform a centralized controller expending equal energy. This is more

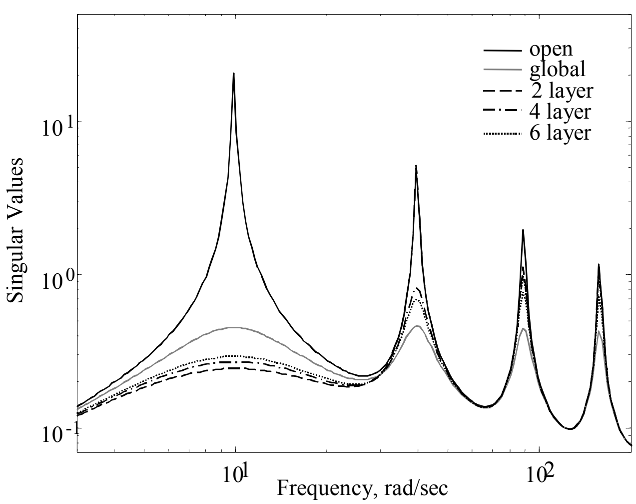
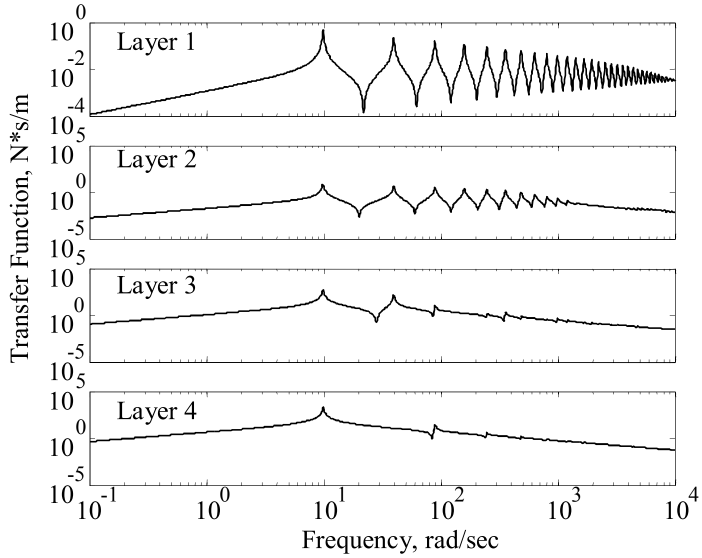


Figure 5 Performance comparison for various vertical hierarchies (zoomed view).

visible in Figure 5 which shows an expanded view of the performance comparison. As can be seen, the centralized controller performs better for all modes above the first mode.

Of particular importance in active structural acoustic control is the ability to target specific modes for attenuation. These are typically the most efficiently radiating or odd modes. This presents an interesting problem in decentralized control because of the need to observe the system globally. Vertical hierarchies are capable of addressing this problem and to act as virtual sensors with selective modal sensitivity. Consider the 4-layer hierarchy of Figure 3. At the highest level of the hierarchy, the agent receives the average of all sensors as the input. This average will be zero for all even modes on the beam since their mode shapes are integer multiple of a full sine wave. So, by increasing the relative performance penalty when designing this layer, one can tune the hierarchy to target the odd modes for attenuation.



This modal selectivity is demonstrated in Figure 6 which

Figure 6 Transfer functions of each layer in the 4-layer hierarchy.

shows the transfer function of one agent from each layer of the 4-layer hierarchy. Note layer 4, the highest layer, is only sensitive to the odd modes. Also note how other layers are sensitive to different sets of modes depending on their configuration.

The ability to tune a hierarchy to attack particular modes is demonstrated in Figure 7. This shows the performance of the 4-layer hierarchy when one of the layers is designed with a performance penalty 10 times larger than the other layers. Note that when layer 4 is designed with a larger performance penalty, only the odd modes are attenuated while the even modes are not. Since all of the hierarch considered here has an even number of agents in each layer, only even modes can be "averaged out" of the agent inputs. However, by designing a hierarchy with an odd number of agents in a layer one could desensitize a particular layer to even modes.

CONCLUSIONS

Hierarchical decentralized control schemes have been considered for active structural acoustic control. The design and performance of various hierarchical arrangements were discussed and the ability of hierarchies to target specific modes for attenuation has been demonstrated. This technique allows for the attenuation of efficiently radiating structural modes through a decentralized observation system.

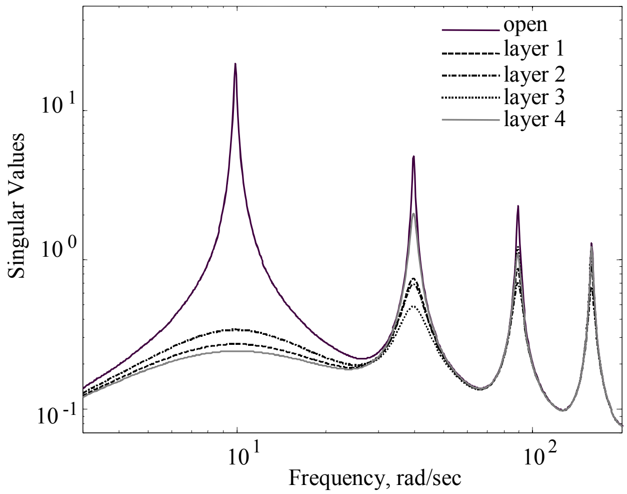


Figure 7 Comparison of performance for 4-layer hierarchy when one layer has a cost weight, α, 10 times that of the other layers.

ACKNOWLEDGMENTS

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